STRENGTH OF MATERIALS

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For the beam presented in Figure below find/calculate/perform:

a) the values of the reactions;

b) the values of shear forces and bending moments for each cross-section of the beam;

c) the maximum values of shear force and bending moment;

d) shear-force and bending-moment diagrams;

e) the allowable dimensions of the cross-section of the beam;

$kg = 150\ MPa$, cross-section No. 4

f) the differential equation of the elastic line;

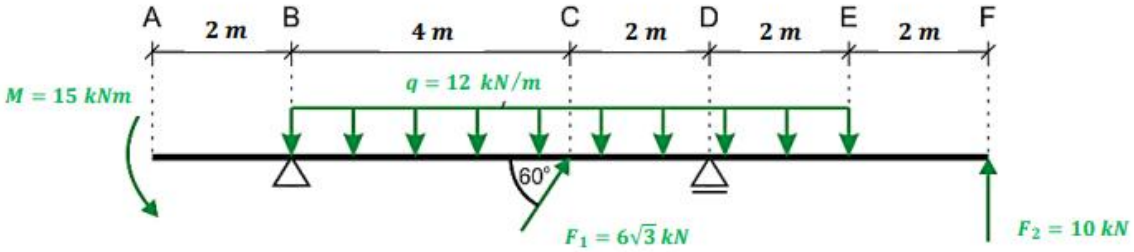
g) the angle of rotation equation;

h) the deflection equation;

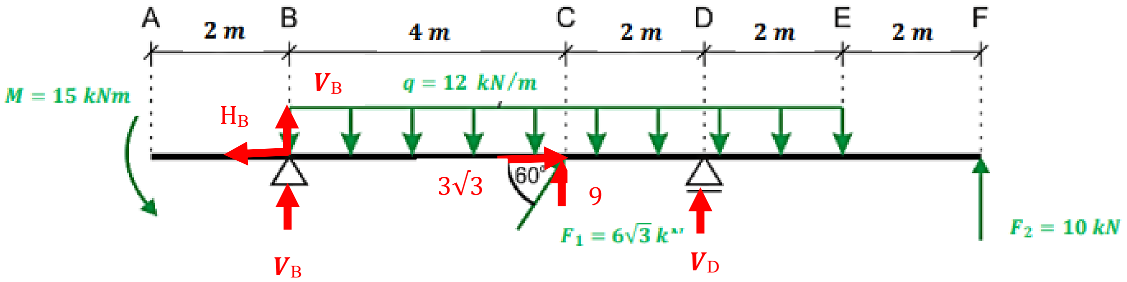
i) the values of C and D parameters;

j) the angle of rotation equation for $x = 6\ m$;

k) the deflection equation for $x = 8\ m$.



a) the values of the reactions;



$$\sum Fx = 0 <=> -HB + 3\sqrt{3}\ kN = 0,$$

$$H_B = 3\sqrt{3}\ kN \text{ (approx.} = 5.20\ kN),$$

$$\sum M_B = 0 <=> -15\ kN + 12\ kN/m \cdot 8\ m \cdot 4\ m - 9\ kN \cdot 4\ m - V_D \cdot 6\ m - 10\ kN \cdot 10\ m = 0,$$

$$V_D = 38.83\ kN,$$

$$V_B + V_D + 9\ kN + 10\ kN - 12\ kN/m \cdot 8\ m = 0,$$

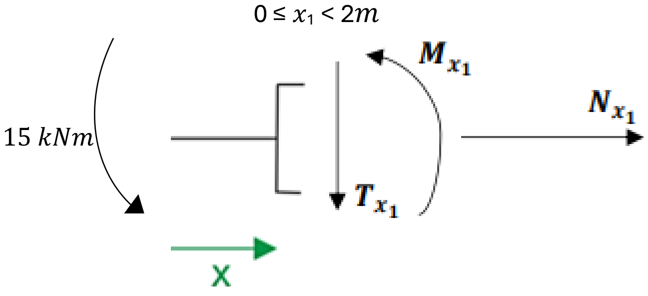
$$V_B + 19 + 38.83 = 96,$$

$$V_B = 38.17\ kN.$$

b) the values of shear forces and bending moments for each cross-section of the beam;

Both the shear force and bending moment in the beam have to be computed in the five different regions. The reason is that there are discontinuities caused by the external loads.

1) Cross-section No. 1:



The values of the bending moment:

$$M_{x1} = 15\ kNm$$

and the expression for shear force is as follows

$$0 - Tx1 = 0$$

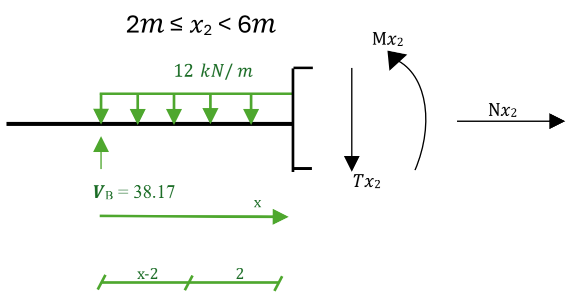
$$Tx1 = 0\ kN.$$

Normal force for the first cross-section:

$$0 - Nx1 = 0,$$

$$Nx1 = 0\ kN.$$

2) Cross-section No. 2:



The expression for bending moment is:

$$38.17 \cdot (x-2) - \frac{12 \cdot (x-2)^2}{2} - Mx_2 = 0,$$

$$Mx_2 = 38.17(x-2) - 6(x-2)^2 \text{-} 15$$

The values of the bending moment:

$$M_{x2=2\ m} = \text{-}15\ kNm\ ,$$

$$M_{x2=6\ m} = 41{,}68\ kNm\ ,$$

and the expression for shear force is as follows

$$38.17 - 12(x-2) - Tx_2 = 0,$$

$$Tx_2 = 38.17 - 12(x-2)\ ,$$

The values of the shear force:

$$T_{x2=2\,m} = 38.17\ kN\ ,$$

$$T_{x2=6\,m} = -9.83\ kN\ ,$$

Normal force for the second cross-section:

$$0 - Nx_2 = 0,$$

$$Nx_2 = 0\ kN.$$

Due to the fact that the function of the shear force changes its sign in the second interval, there must be an extreme bending moment in it. Thus, in order to determine the cross-section in which the function Mx_2 reaches its extreme, we have to compare the function Tx_2 to zero. Because

$$\frac{\mathrm{dMx2}}{\mathrm{dx}} = T_{x2} = 38.17 - 12(x-2) = 0,$$

$$X_0 = 5.1808\ m.$$

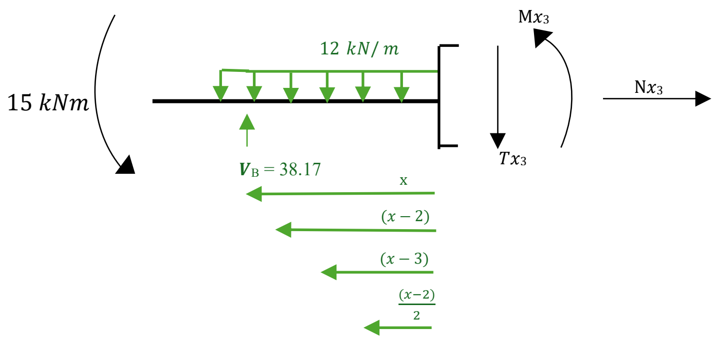
The maximum value of the bending moment in the second cross-section:

$$M_{x2=x0} = 38.17(x-2) - 6(x-2)^2 - 15$$

$$M_{x2=x0} = 45.70\ kNm.$$

3) Cross-section No. 3:

$$6\ m \leq x_3 < 8m$$



The expression for bending moment is:

$$38.17 \cdot (x-2) - 15 - 6 \cdot (x-2)^2 + 9 \cdot (x-6) - Mx_3 = 0$$

$$Mx_3 = 38.17 \cdot (x-2) - 15 - 6 \cdot (x-2)^2 + 9 \cdot (x-6).$$

The values of the bending moment:

$$M_{x3=6\,m} = 41.68\ kNm\ ,$$

$$M_{x3=8\,m} = 16\ kNm\ ,$$

and the expression for shear force is as follows

$$38.17 + 9 - 12 \cdot (x - 3) + Tx3 = 0,$$

$$Tx_3 = 12 \cdot (x - 3) - 38.17 - 9\ ,$$

The values of the shear force:

$$T_{x3=6\,m} = -\ 0.83\ kN\ ,$$

$$T_{x3=8\,m} = -24.83\ kN\ .$$

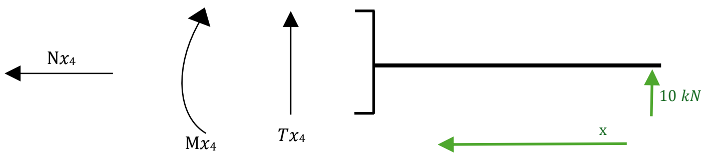
Normal force for the third cross-section:

$$5.196 + Nx_3 = 0,$$

$$Nx_3 = -\ 5.196\ kN\ .$$

4) Cross-section No. 4 (solution from the right end of the beam):

$$2m > x_4 \geq 0m$$



The expression for bending moment is:

$$10 \cdot x - Mx4 = 0,$$

$$Mx4 = 10 \cdot x.$$

The values of the bending moment:

$$M_{x4=0\,m} = 0\ kNm\ ,$$

$$M_{x4=2\,m} = 20\ kNm\ ,$$

and the expression for shear force is as follows

$$-10 + Tx_4 = 0,$$

$$Tx_4 = -\ 10\ kN.$$

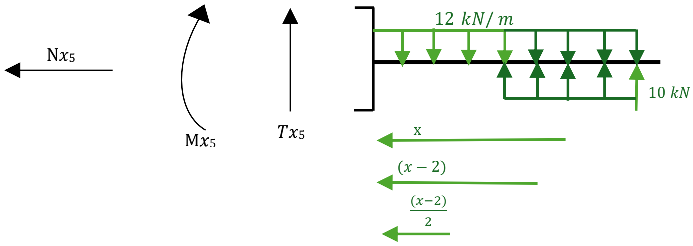
Normal force for the fourth cross-section:

$$0 + Nx_4 = 0,$$

$$Nx_4 = 0\ kN.$$

5) Cross-section No. 5 (solution from the right end of the beam):

$$2m \le x_5 < 4m$$



The expression for bending moment is:

$$10 \cdot x - 6 \cdot (x-2)^2 + Mx_5 = 0,$$

$$Mx_5 = 6(x-2)^2 + 10x.$$

The values of the bending moment:

$$M_{x5=2\,m} = 20\ kNm\,,$$

$$M_{x5=4\,m} = 16\ \ kNm\,,$$

and the expression for shear force is as follows

$$10 - 12 \cdot (x-2) + Tx_5 = 0,$$

$$Tx_5 = 12(x-2) - 10.$$

The values of the shear force:

$$T_{x5=2\,m} = -\,10\ kN\,,$$

$$T_{x5=4\,m} = 14\ kN\,.$$

Normal force for the fifth cross-section:

$$0 + Nx_5 = 0,$$

$$Nx_5 = 0\ kN\,.$$

Due to the fact that the function of the shear force changes its sign in the fifth interval, there must be an extreme bending moment in it. Thus, in order to determine the cross-section in which the function Mx_5 reaches its extreme, we have to compare the function Tx_5 to zero. Because

$$\frac{\mathrm{dMx5}}{\mathrm{dx}} = T_{x5} = -\,10 + 12(x-2) = 0,$$

$$X_0 = 2.8333\ m.$$

The maximum value of the bending moment in the fifth cross-section:

$$M_{x5=x0} = -\,6(x-2)^2 + 10 \cdot x\,,$$

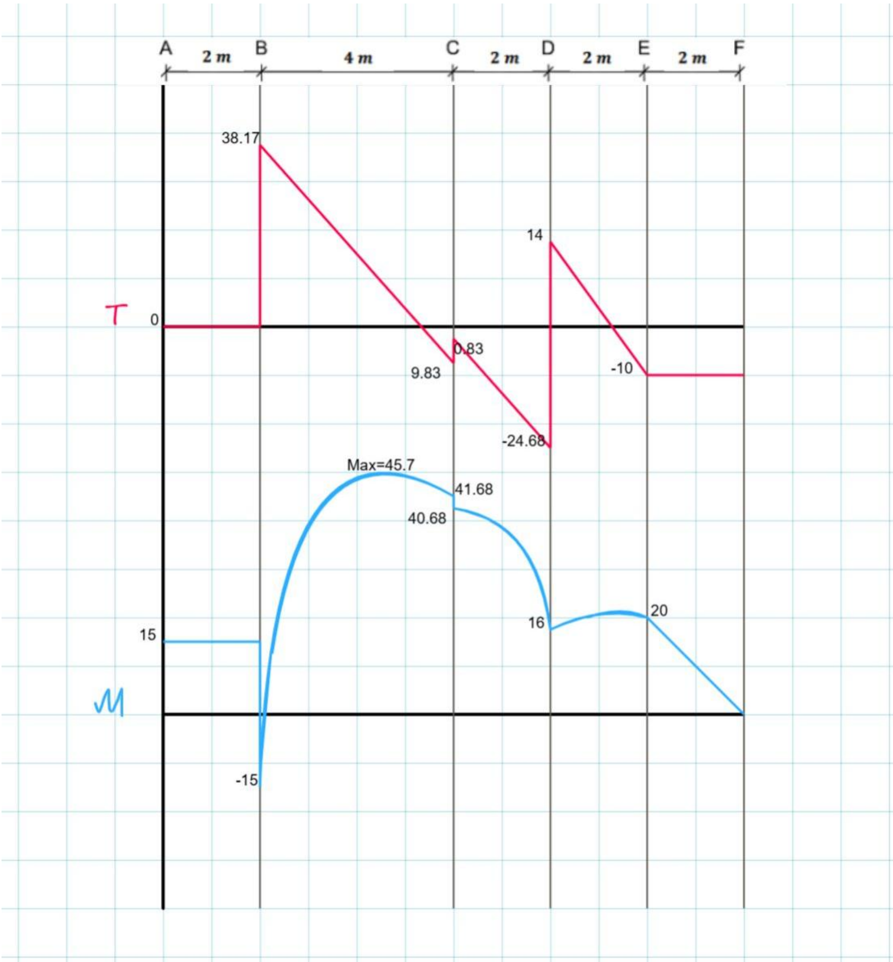
$$M_{x5=x0} = 24{,}167\ kNm.$$

c) the maximum values of shear force and bending moment;

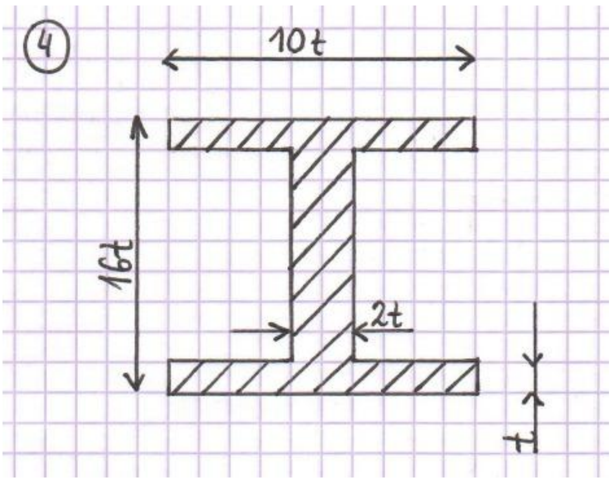
$$T_{\max} = 38.17\ kN,$$

$$M_{\max} = 45.70\ kNm.$$

d) shear-force and bending-moment diagrams (+ normal-force diagram);



e) the allowable dimensions of the cross-section of the beam;



Kg =150 MPa (kg – allowable stresses for bending)

The moment of inertia:

(Cross-section no. 4.)

$$I_{x_c} = \frac{10t \cdot 16t^3}{12} - \frac{8t \cdot 14t^3}{12} = 1584t^4.$$

The section modulus:

$$\boldsymbol{\omega} = \frac{\boldsymbol{I_{xc}}}{\boldsymbol{Ymax}} = \frac{\mathbf{1584t^4}}{\mathbf{8t}} = 198t^3$$

The strength condition:

$$\left|\frac{Mmax}{\omega}\right| \leq Kg$$

$$\frac{45{,}70\ KNm}{198t^3} \leq 150\ MPa$$

$$\frac{45{,}70 \cdot 10^6\ Nmm}{198t^3} \leq 150\frac{N}{mm^2}$$

$$45{,}70 \cdot 10^6\ Nmm \leq 150\frac{N}{mm^2} \cdot 198t^3$$

$$t\ \geq 11{,}54\ mm$$

$$t\ = 12\ mm$$

Normal Stresses:

$$\sigma = \frac{Mmax}{\omega}$$

$$\frac{45{,}70\ KNm}{198t^3} = \frac{45{,}70 \cdot 10^6\ Nmm}{198\ (12)^3} = 133{,}56\ MPa$$

f) the differential equation of the elastic line;

Bending moment for the whole beam:

$$M_x = -15x_0 + V_B(x-2) - (F_1 \cdot \cos 60) - \frac{12(x-2)^2}{2} + V_D\,(x-8) + \frac{12(x-10)^2}{2},$$

$$M_x = -15x_0 + 38.17(x-2) - 9(x-6) - 6(x-2)^2 + 38.83(x-8) + 6(x-10)^2,$$

$$EI\frac{d^2y}{dx^2} = 15x_0 - 38.17(x-2) - 9(x-6) - 6(x-2)^2 + 38.83(x-8) + 6(x-10)^2,$$

g) the angle of rotation equation;

$$EI\frac{dy}{dx} = 15x - \frac{38.17(x-2)^2}{2} - \frac{9(x-6)^2}{2} + \frac{6(x-2)^3}{3} - \frac{38.83(x-8)^2}{2} - \frac{6(x-10)^3}{3} + C.$$

h) the deflection equation;

$$EIy = \frac{15x^2}{2} - \frac{38.17(x-2)^3}{6} - \frac{9(x-6)^3}{6} + \frac{6(x-2)^4}{12} - \frac{38.83(x-8)^3}{6} - \frac{6(x-10)^4}{12} + C \cdot x + D.$$

i) the values of *C* and *D* parameters;

The deflection is equal to 0:

For $x = 2\ m$:

$$15\ kNm\frac{(2\ m)^2}{2} + \mathrm{C} \cdot 2\ m + \mathrm{D} = 0,$$

$$30\ kNm^3 + \mathrm{C} \cdot 2\ m + \mathrm{D} = 0,$$

For $x = 8\ m$:

$$15\ kNm\frac{(8\ m)^2}{2} - \frac{38{,}167\ kN(8\ m - 2\ m)^3}{6} + \frac{6\frac{kN}{m}(8\ m - 2\ m)^4}{12} - \frac{9\ kN(8\ m - 6\ m)^3}{6} + \mathrm{C} \cdot 8m + \mathrm{D} = 0$$

$$480\ kNm^3 - 1374\ kNm^3 - 12\ kNm^3 + 648.0\ kNm^3 + \mathrm{C} \cdot 8\ m + \mathrm{D} = 0,$$

$$-258\ kNm^3 + \mathrm{C} \cdot 8\ m + \mathrm{D} = 0,$$

$$\begin{cases} 30 + 2C + D = 0 \\ -258 + 8C + D = 0 \end{cases}$$

$$\mathrm{C} = 48\ kNm^3$$

$$\mathrm{D} = -126\ kNm^3$$

j) the angle of rotation equation for *x* = 6 *m*;

$$\frac{dy}{dx} = \frac{1}{EI}\left[15\ kNm \cdot 6\ m - \frac{38{,}167\ kN(6\ m - 2\ m)^2}{2} + \frac{6\frac{kN}{m}(6\ m - 2\ m)^3}{3} + 48\ kNm^2\right],$$

$$\frac{dy}{dx} = \frac{1}{EI}[90kNm^2 - 305{,}33kNm^2 + 128kNm^2 + 48kNm^2],$$

$$\frac{dy}{dx} = -\frac{39\ \mathrm{kNm^2}}{EI}$$

k) the deflection equation for *x* = 8 *m*;

$$y = \frac{1}{EI}[15\ kNm\frac{(8\ m)^2}{2} - \frac{38{,}167\ kN(8\ m - 2\ m)^3}{6} + \frac{6\ \frac{kN}{m}(8\ m - 2\ m)^4}{12} - \frac{9\ kN(8\ m - 6\ m)^3}{6} + C \cdot 8\ m + D] = 0,$$

$$y = \frac{1}{EI}[480\ kNm^3 - 1374\ kNm^3 + 648\ kNm^3 - 12\ kNm^3 + C \cdot 8\ m + D = 0,$$

$$-258\ kNm^3 + C \cdot 8\ m + D]$$

$$y = \frac{0kNm^3}{EI}$$